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PUMPING TEST DATA INTERPRETATION AND DRIFT INFLOW ASSESSMENT BY FINITE-DIFFERENCE RADIAL MODELLING

by

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ABSTRACT

A digital radial (r-z-t) finite-difference model for the simulation of constant-discharge pumping-tests in multilayered ground is described. The model is particularly useful for the interpretation of multiport piezometer data since the drawdown at each port can be independently simulated on the r-z-t grid. Unlike conventional methods of pumping-test interpretation, where flow and ground properties are usually averaged over one or two layers, the radial (r-z-t) model allows a detailed approximation of the vertical variation in ground hydraulic properties.

A case study is described in which the radial (r-z-t) model is applied to the interpretation of multiport piezometer data from two pumping-tests carried out in a site investigation for a proposed new surface drift. The resulting permeability distribution is then used as input data to model inflows to the proposed drift under construction. Three-dimensional inflows to a short length of unlined drift open section are approximated using steady-state radial (r-z) flow models. Use of the numerical modelling techniques result in a better understanding of the groundwater system than that obtained from conventional analytical techniques.

INTRODUCTION

In recent years the use of multiport piezometers to record drawdown data during pumping-tests has become more common, particularly in site investigations for major sub-surface engineering projects. Multi-port piezometers allow the acquisition of data from several ports (up to ten is common) at different levels in one borehole. The advantages over a conventional nested piezometer include the smaller diameter borehole required and the speed of installation.

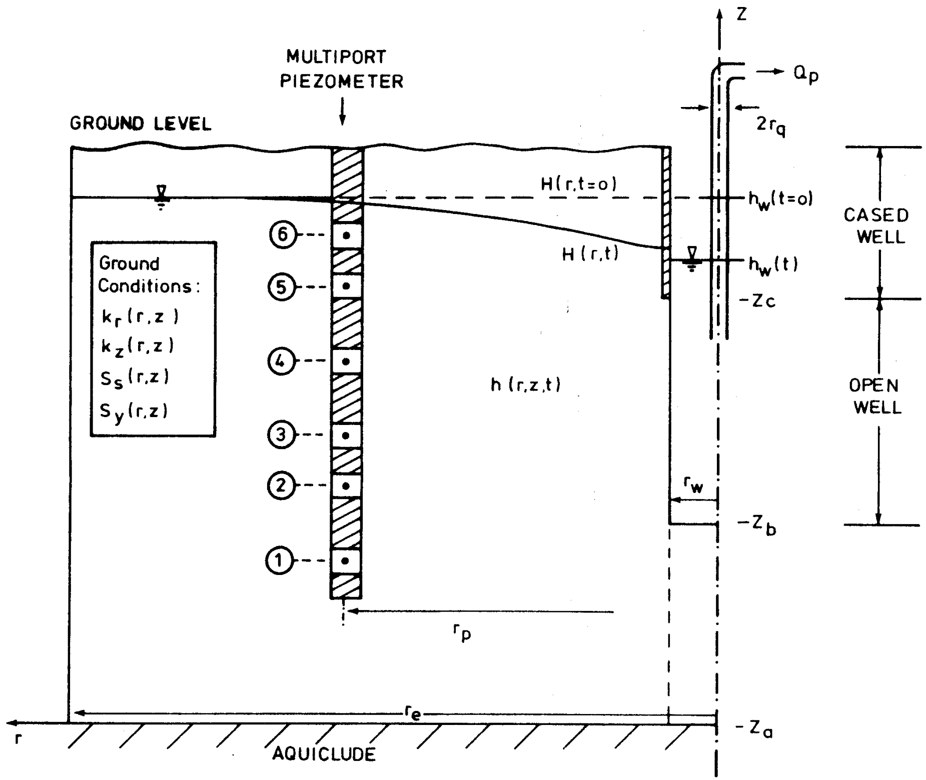


Figure 1. Definition sketch of pumping-test in multilayered ground.

Multi-port piezometers are potentially most useful where detailed information is required on groundwater conditions in multilayered ground. If a surface shaft or drift is to be sunk, for example, it is important to be able to predict groundwater inflows with depth on a detailed scale. This in turn demands a detailed knowledge of the permeability distribution. Pumping-tests carried out with one or more multiport piezometers can provide the necessary raw drawdown data. However, the techniques available for interpreting these data are generally unsatisfactory.

The interpretation of pumping-test data is conventionally carried out using analytical techniques (Kruseman and de Ridder, 1970) and one or two-layer radial flow models (Rushton and Redshaw, 1979). Neither approach is ideal since both rely on the averaging of groundwater head and permeability conditions in a vertical direction over one or two model layers, and little detailed information is gained on the vertical variation of ground hydraulic properties and in particular on vertical permeability itself. The value of installing a multiport piezometer can only be properly realized if the method of pumping-test data interpretation allows the independent simulation of drawdown at each piezometer port. This requires a full radial (r-z-t) model, which in conjunction with geological and geophysical borehole logs, can provide a detailed approximation of the permeability distribution in multilayered ground.

This paper initially reports on the development and application of a digital radial (r-z-t) finite-difference model for the interpretation of multiport piezometer pumping-test data. A case study is described in which the model is applied to the interpretation of multiport data from two pumping-tests carried in a site investigation for a proposed new surface drift. The study illustrates the benefits of applying the radial (r-z-t) model where ground conditions are heterogeneous and multilayered.

Once the three-dimensional permeability distribution over the drift site has been approximated, it is then possible to assess the magnitude of groundwater inflows to the drift during construction. The technique adopted in the case study described is based on the approximation of inflows to a short length of open drift by steady-state radial (r-z) inflow modelling (Edwards, 1987; Lloyd and Edwards, 1988).

MATHEMATICAL FORMULATION OF PUMPING-TEST PROBLEM

A pumping-test involving the use of a multiport piezometer is shown schematically in Figure 1. The well is cased from ground level to an elevation Z_c, and then screened or left as open-hole to an elevation Z_b. The multiport piezometer is installed at a distance r_p from the well and, in the example shown, includes six measuring ports. Water is pumped at a constant discharge rate Q_p and flow towards the well is unconfined. Assuming Darcian flow and axisymmetric conditions about the well, saturated flow towards the well during the pumping-test is governed by the following equations (Bear, 1972; Gambolati, 1976).

The radial (r-z-t) flow equation is:

$$\frac{\partial}{\partial r}\frac{k_r \partial h}{\partial r} + \frac{k_r}{r}\frac{\partial h}{\partial r} + \frac{\partial}{\partial z}\frac{k_z \partial h}{\partial z} = S_s \frac{\partial h}{\partial t} \qquad (1)$$

where k_r and k_z are radial and vertical permeability, and S_s is the specific storage coefficient.

$$h(r, z, 0) = H(r, 0) = h_W(0) = \text{constant} \qquad (2)$$

where $H(r, t)$ is the free surface and $h_W(t)$ is the head in the well.

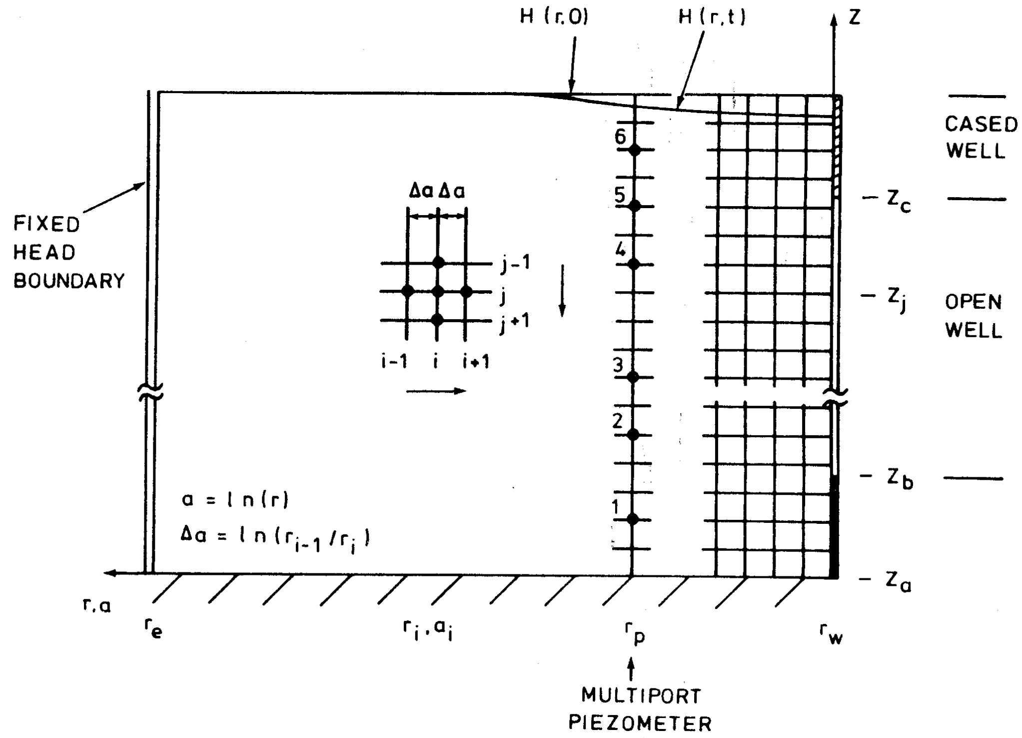


Figure 2. Definition sketch of radial (r-z-t) finite-difference grid indicating allocation of six piezometer ports to grid nodes.

The boundary conditions to the pumping-test problem are as follows. An impermeable horizon forms the base to the section of ground considered:

$$\frac{\partial h}{\partial z}(r, z_a, t) = 0 \qquad 0 < r < r_e \qquad (3)$$

On the inner boundary, assuming that the casing is impermeable:

$$\frac{\partial h}{r}(r_w, z, t) = 0 \qquad z_a < z < z_b; \quad z_c < z < H(r_w, t) \qquad (4)$$

$$h(r_w, z, t) = h_w(t) \qquad z_b < z < z_c \qquad (5)$$

Equation (5) assumes that the head on the open section of the well is constant with depth but variable in time as the water level in the well is lowered. Frictional effects on flow in the well are therefore ignored.

The boundary condition on the outer radial boundary of the model ($r=r_e$) is a fixed-head condition h (r_e, z, 0). This constitutes the "radius of influence" of the pumped well Ignoring infiltration, and following Singh (1976), the non-linear condition on the free surface H(r, t) can be expressed as:

$$\frac{\partial H}{\partial t} = v_z - v_r \frac{(\partial H)}{(\partial r)} \qquad (6)$$

where the specific velocities v_r and v_z at the free surface are defined as:

$$v_r = \frac{-k_r}{S_y} \frac{(1 - \partial h)}{\partial z} \frac{\partial H}{\partial r}$$

$$v_z = \frac{-k_z}{S_y} \frac{\partial h}{\partial z} \qquad (7)$$

where S_y is the specific yield.

The term $\frac{\partial h}{\partial z}$ represents the vertical groundwater head gradient at the free surface. The second condition on the free surface (on which pressure is assumed to be atmospheric and zero) is:

$$h(r, H(r, t), t) = H(r, t) \qquad (8)$$

The final boundary condition relates to the discharge condition on the open well ($Z_b < Z < Z_c$) At time t the discharge Q_g from the ground to the well is given by:

$$Q_g(t) = 2\tau\ r_w \int_{z_b}^{z_e} k_r(r_w,z)\ \frac{\partial h}{\partial r}\ (r_w,z,t)\ dz$$

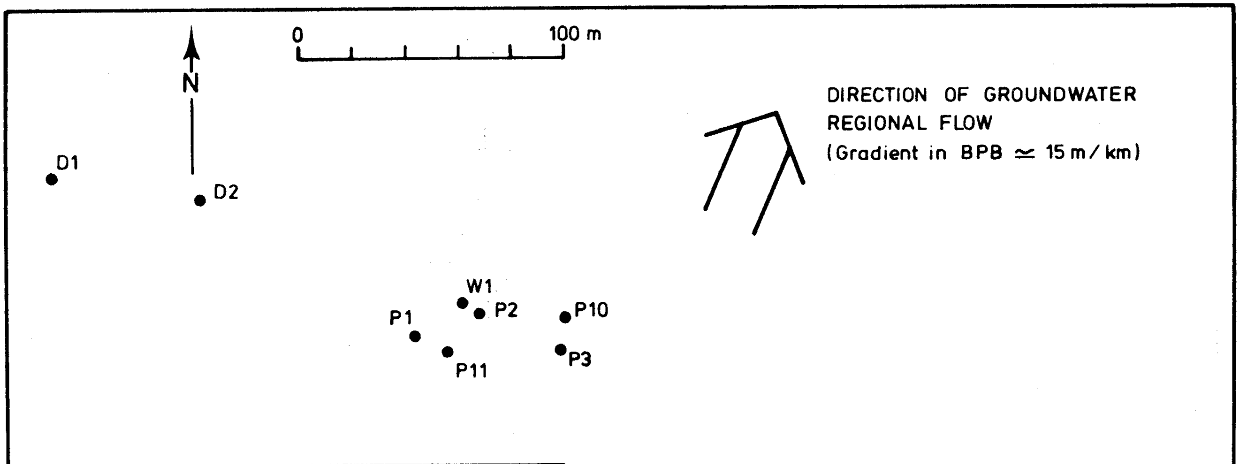
However, the discharge Q_p at surface is composed of water drawn both from the ground and from storage in the well. Assuming that the rising main has an outside radius of r_q, the discharge condition may be expressed as:

Equations (1) - (6), (8) and (10) define the problem of simulating a pumping-test in multilayered ground. The aim in pumping-test data interpretation is to obtain h (r, z, t) and h_w (t) such that the model head functions at the multiport piezometer ports, and the well, match the field data to an acceptable degree of accuracy. Since k_r (r, z), k_z (r, z), S_s (r, z) and S_y (r, z) are unknowns, this involves the examination of a series of solutions based on trial permeability and storage distributions in order to obtain a 'best-fit' solution.

FINITE-DIFFERENCE MODEL FOR PUMPING-TEST DATA INTERPRETATION

The digital finite-difference radial (r-z-t) model developed for the interpretation of multiport pumping-test data is capable of simulating non-steady-state saturated Darcian flow towards a well discharging at a constant rate. The model can simulate pumping-tests in multilayered ground, in which each layer can display anisotropy. Time is discretised in logarithmcal expanding time-steps, and implicit (backward-difference) numerical solutions for h (r, z, t) and h_W (t) are obtained at the end of each time-step (Rushton and Redshaw, 1979). Solutions are formed on a finite-differnece grid (Figure 2) in which the vertical grid spacing may be regular or irregular, and the radial grid spacing is regular in terms of a, where a = ln (r). This transformation is applied to the governing flow equation and reduces the truncation error in the numerical approximation (Rushton and Redshaw, 1979). The multiport piezometer ports are assigned to the nearest grid nodes. Suitable grid design can minimise any errors in model port location. The Strongly Implicit Procedure (Stone, 1968) is used to solve the sets of finite-difference equations.

The head in the well h_W at one time-step cannot be directly calculated from the solution at the previous time-step. It is therefore necessary to employ an outer iterating scheme in which successive trial solutions for h (r, z, t) and h,., (t) are evaluated until the flow equation (eqn. 1) and the open well boundary condition (eqn. 10) are simultaneously satisfied to an acceptable degree of accuracy. Neuman and Witherspoon (1971) describe



GEOLOGICAL FORMATION	AVERAGE DEPTH (m bgl)
SUPERFICIAL DEPOSITS (SU)	0–20
BUNTER SANDSTONE (BS)	20–40
BUNTER PEBBLE BEDS (BPB)	40–115
LITTLEWORTH SANDSTONE (LS)	115–148
COAL MEASURES (CM)	below 148m

Figure 3. Plan of pumping-test site indicating borehole positions and general geological succession.

this technique, although they use the finite-element method to solve the flow equations. The model takes well storage directly into account through the open well boundary condition (eqn. 10). Based on field data, well losses can be approximated in the model by reducing the model radial permeability immediately adjacent to the well (Rushton and Redshaw, 1979).

Flow towards the well can be confined or unconfined; in the latter case the movement of the free surface H (r, t) in each time-step is predicted in an explicit (forward-difference) manner (Singh, 1976). Movement of the free surface results in the truncation, and therefore requires the re- definition, of the finite-difference grid at each time-step. Since the flow equation (eqn. 1) is solved implicitly, and the free surface boundary condition (eqn. 6) explicitly. the two equations are never simultaneously satisfied. Solution stability and accuracy are therefore dependent on the careful selection of a relatively small maximum time-step (Guvanasen and Volker, 1980).

No analytical solutions are available to test all features of the radial (r-z-t) model. However, model solutions to simple test problems were compared with various reported analytical and numerical solutions to demonstrate that the model solved the basic radial flow equations correctly. In all cases, favourable comparisons were achieved (eg less than 5% error in discharge rates) that indicated that the radial (r-z-t) model was performing satisfactorily.

CASE STUDY: PUMPING-TEST DATA INTERPRETATION

GENERAL DETAILS

The radial (r-z-t) model has been applied in the interpretation of pumping-test data obtained during an extensive site investigation to examine the feasibility of a proposed new surface drift. An initial pumping-test was carried out in well Wl on the site plan in Figure 3. Detailed drawdown data were recovered from multiport piezometers installed in boreholes Pl and P2, and conventional single or double standpipe piezometers installed in boreholes Dl, D2, Pl 1, P3, P10, and P8. The piezometer open intervals and a schematic geological sections (WNW- ESE) are shown in Figure 4. Water was taken from Wl over a 136m test interval, extending through three Triassic formations: the Bunter Sandstone; the Bunter Pebble Beds; and the Littleworth Sandstone. The site investigation indicated that all three formations were extremely variable with significant lateral and vertical variations in lithology and hydraulic properties.

An initial interpretation of the Wl pumping-test data, using analytical type curves and single or double-layer radial models, could not provide detailed information on the multilayered permeability conditions in the Triassic strata. Difficulties were experienced in obtaining reasonable matches between field and simulated data. In view of the importance of the permeability and head parameters in the drift inflow calculations, it was decided to supplement the conventional interpretation with a new interpretation based on the (r-z-t) model.

Two constraints were placed on the analysis in order to reduce the number of possible variables in the simulation process. First, the correlation between field and modelled drawdown data was attempted only at multiport piezometer Pl. Second, other than for the approximation of well losses close to Wl, it was decided that each model permeability layer should display a constant radial and vertical permeability with respect to radial distance from W1. Natural axisymmetric variations in hydraulic properties about a well are in any case unlikely.

The type of multiport piezometer used in this site investigation suffers from the disadvantage that high-frequency pressure readings are difficult to obtain at all the piezometer ports on the string. This is due to the mechanical nature of the pressure

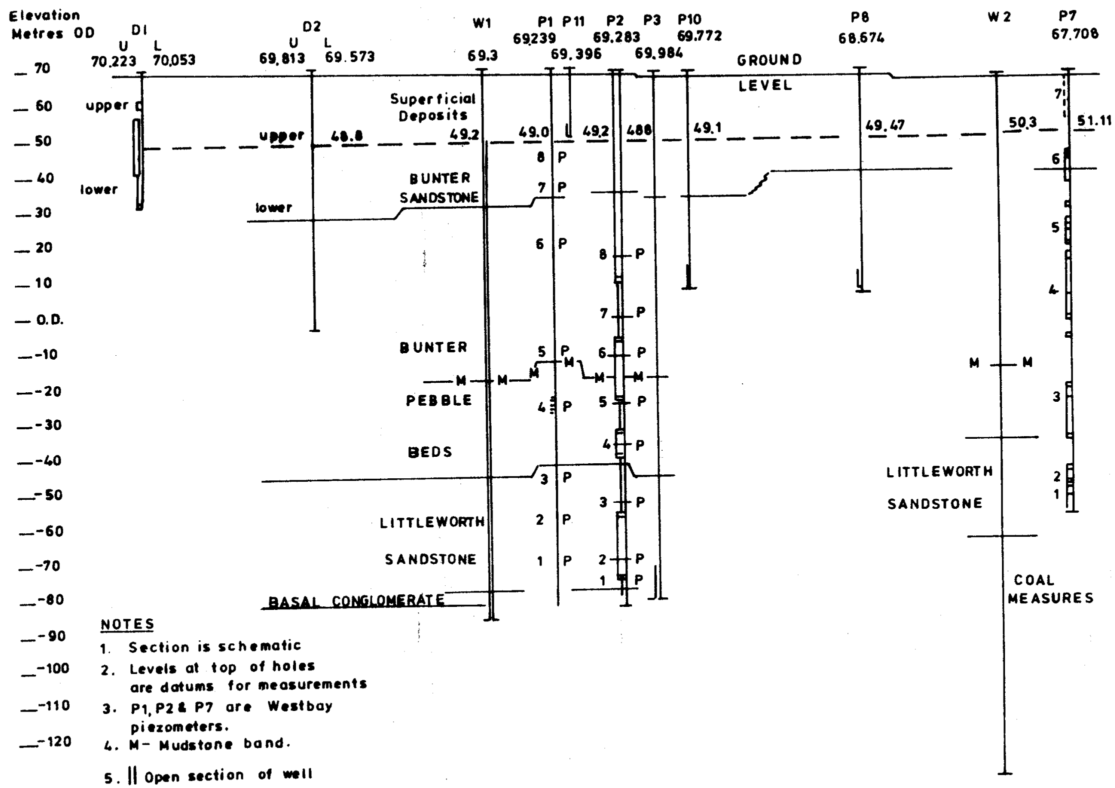


Figure 4. Piezometer response zones and schematic geological cross-section.

measurement device. The practice during the early part of a pumping-test, when the collection of high-frequency data from all ports would be desirable, is therefore to leave the measuring device locked on to one port. In the Wl pumping-test early data were obtained in this way from Pl port 6; data from all ports were collected from 100 minutes after the start of pumping.

The Wl pumping-test was carried out for five days at a constant discharge rate of 976m^3/d. The pump was installed at a depth of 10m below ground level within the cased well. Borehole Pl was constructed 26.3m from Wl.

GEOLOGY

The geological succession in the site area is indicated on Figures 3 and 4, and consists of alluvial sand and gravel deposits unconformably overlying Triassic sandstones and conglomerates. The strata rise gently to the south-east. No evidence of major faulting was found during the site investigation. Cores recovered from the boreholes indicate that the Triassic strata are extremely variable in terms of lithology, cementation, and lateral continuity. The Bunter Sandstone is a medium to coarse grained sandstone, which in common with the upper 10m of the Bunter Pebble Beds, is highly weathered and fractured. The Bunter Pebble Beds consist of conglomerates and sandstones, with some thin (less than 1m) mudstone bands, including one prominent band (0.8m thick) at an elevation of approximately -20m OD in the region of Wl and Pl (Figure 4). The Littleworth Sandstone is a medium to coarse grained sandstone with a well-developed joint system. The Coal Measures consist mainly of mudstones, which are weathered to clay at the base of the Triassic Strata.

HYDROGEOLOGY

There is a regional groundwater head gradient towards the north east of 15m/km in the Bunter Pebble Beds, and 2.5m/km in the superficial deposits. Based on data recorded at all available piezometers, an initial groundwater head distribution before the start of the Wl pumping-test was estimated (Figure 5). The geological system is approximated as being horizontally layered for the purposes of pumping-test simulation. Well Wl was constructed before the multiport piezometers had been installed in boreholes Pl or P2, thereby providing an efficient hydraulic connection between the different formations. This allowed flow from the Bunter Pebble Beds and the Littleworth Sandstones to the Bunter Sandstone and the superficial deposits, and disturbed the natural groundwater head conditions in the vicinity of Wl.

The natural groundwater head conditions are probably best displayed at borehole P7. This indicates that the water-table in the superficial deposits lies at approximately 3m below ground level. Groundwater head increases with depth, reaching a peak value towards the base of the Bunter Pebble Beds, and then decreases slightly in the Littleworth Sandstone. It is probably that groundwater head variation with depth is controlled by the multilayered character of the Triassic formations, with horizons of low permeability restricting vertical seepage.

The site investigation included extensive single-borehole permeability testing in boreholes Dl, D2, Wl and W2 prior to the pumping-test in Wl. A wide range of results were obtained, indicating highly variable permeability conditions. The degree of hydraulic continuity between the Bunter Pebble Beds and the Bunter Sandstone appears to be limited, as suggested by the relatively large vertical head gradient in the upper part of the Bunter Pebble Beds (Figure 5).

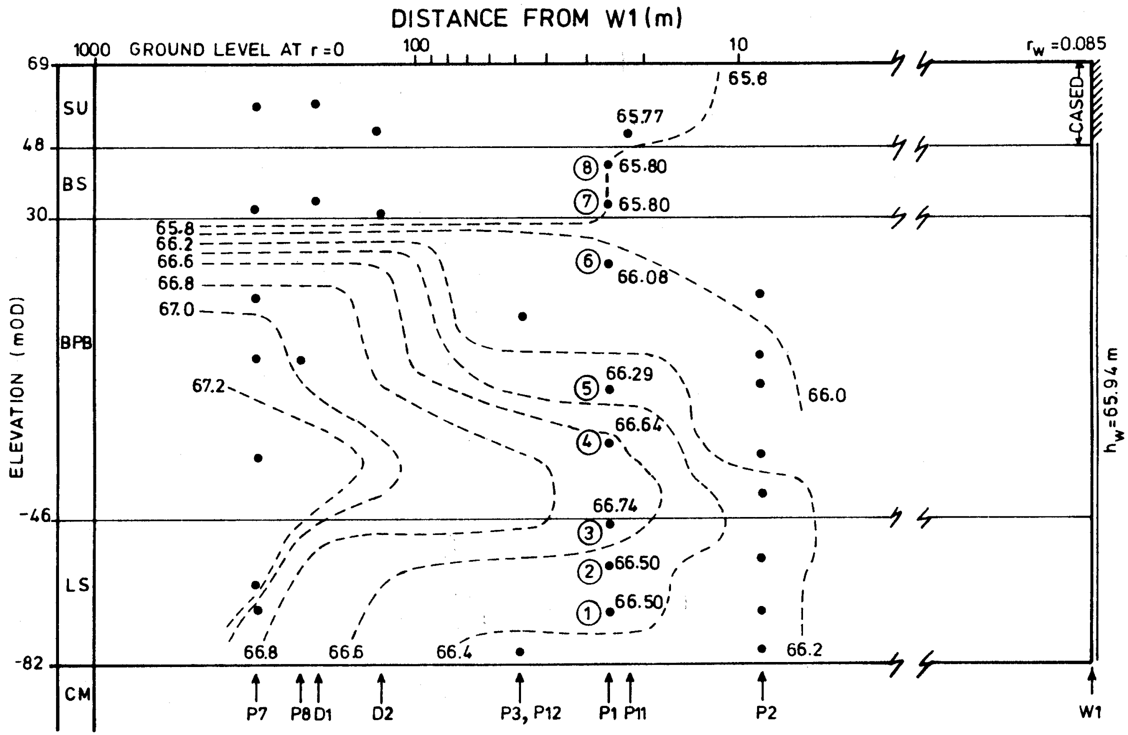


Figure 5. Pumping-test in W1: Initial groundwater head (m OD) and piezometry. Solid circles indicate multipoint piezometer ports (P1, P2, P7) and central points of piezometer response zones.

MODELLING DETAILS

The radial (r-z-t) model was prepared in the following manner. Three constraints applied to the design of a suitable finite-difference grid: the accurate representation of the well radius (r_W = 0.085m), the radial distance from Wl to the multiport piezometer installed in Pl (r_p = 26.3m), and the piezometer port elevations. The selected grid was based on a regular logarithmic radial spacing of six intervals between 0.027, 0.27, 27, 270, and 2700m. This allowed the model well radius to be set at 0.085m, and Pl at a radius of 27m. The outer radial boundary was set initially at 2700m. A regular vertical grid spacing of 5m was employed between elevations - 85m OD and 60m OD (Figure 5). The initial free surface position was defined at 65.8m OD, and the uppermost vertical grid interval therefore extended between 60m OD and 65.8m OD. The piezometer ports in Pl were assigned to the nearest grid notes on the grid vertical r = 27m. A horizontal no-flow boundary was imposed at the base of the Triassic formations since little flow to the well was expected from the Coal Measure mudstones.

A step-drawdown test was carried out on Wl prior to the 5-day pumping-test. From an analysis of the data (Cooper and Jacob, 1946) it was estimated that approximately 25% of the drawdown in Wl during the pumping-test was due to non-linear well losses. As a first attempt to represent well losses, the model permeability values between the open well boundary nodes and the radially adjacent modes were multiplied by a factor of 0.75.

No difficulties were encountered during the simulation of the Wl pumping-test with respect to solution stability and the free surface. Field data indicated that the free surface fell only by approximately 0.15m, compared to a well drawdown of 2.50m. Once this had been simulated, it became clear that with a vertical grid spacing of 5.8m, this fall in free surface height was too small to affect solution stability

RESULTS FOR WELL Wl

The best match between field and model drawdown data at the Pl ports is presented in Figure 6. The corresponding permeability distribution is shown in Figure 7. A specific storage coefficient of 10^{-6}/m led to the most accurate model fit against the limited non-steady-state drawdown data from Pl port 6 (Figure 6) and was applied uniformly throughout the modelled domain. A specific yield of 0.01 was used. Figure 7 also demonstrates the close correlation between the modelled and field groundwater head distribution after 3.86 days of the pumping-test. Comparing steady-state drawdown data, the fit between modelled and field data is generally good, although it was difficult to achieve an accurate match at Pl ports 3 and 5.

The modelled permeability distribution includes ground of very low vertical permeability towards the top of the Bunter Pebble Beds. This zone effectively separates the Bunter Sandstone and the superficial deposits hydraulically from the lower formations. Very high permeability zones exist in the Littleworth Sandstone; the model shows that these zones supplied approximately 75% of the discharge during the Wl pumping-test. The correlation between the model and field head distribution after 3.86 days is good. This is based on field and model data from all the piezometer ports and response zones, and would appear to confirm that a reasonable permeability distribution has been estimated. It was possible to simulate the steady-state well drawdown exactly (Figure 7) by adjusting the well loss factor from 0.75 to 0.8. The outer radial fixed-head boundary was set at 853m in order to simulate the steady conditions.

In non-steady-state terms, the fit at Pl port 6 is not particularly convincing. Without more data. it was not appropriate to attempt to estimate a more detailed storage

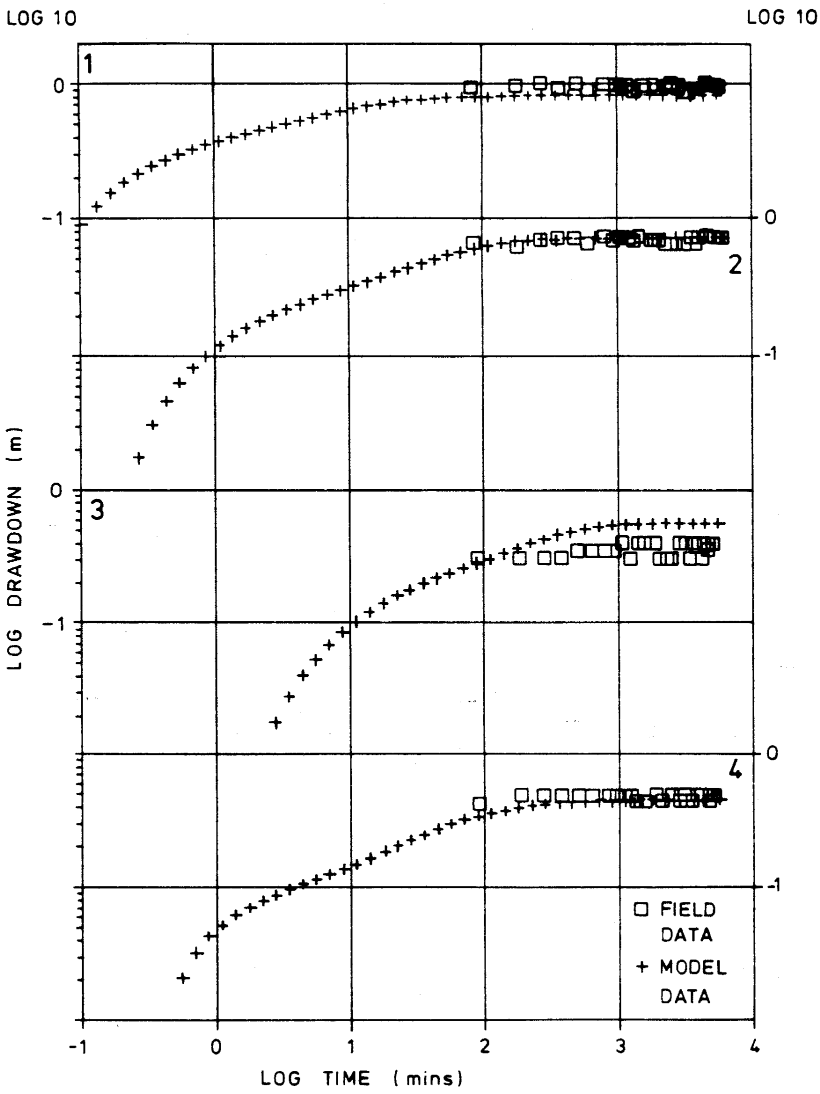


Figure 6(a) Field and model drawdowns at Pl (a) ports 1-4 .

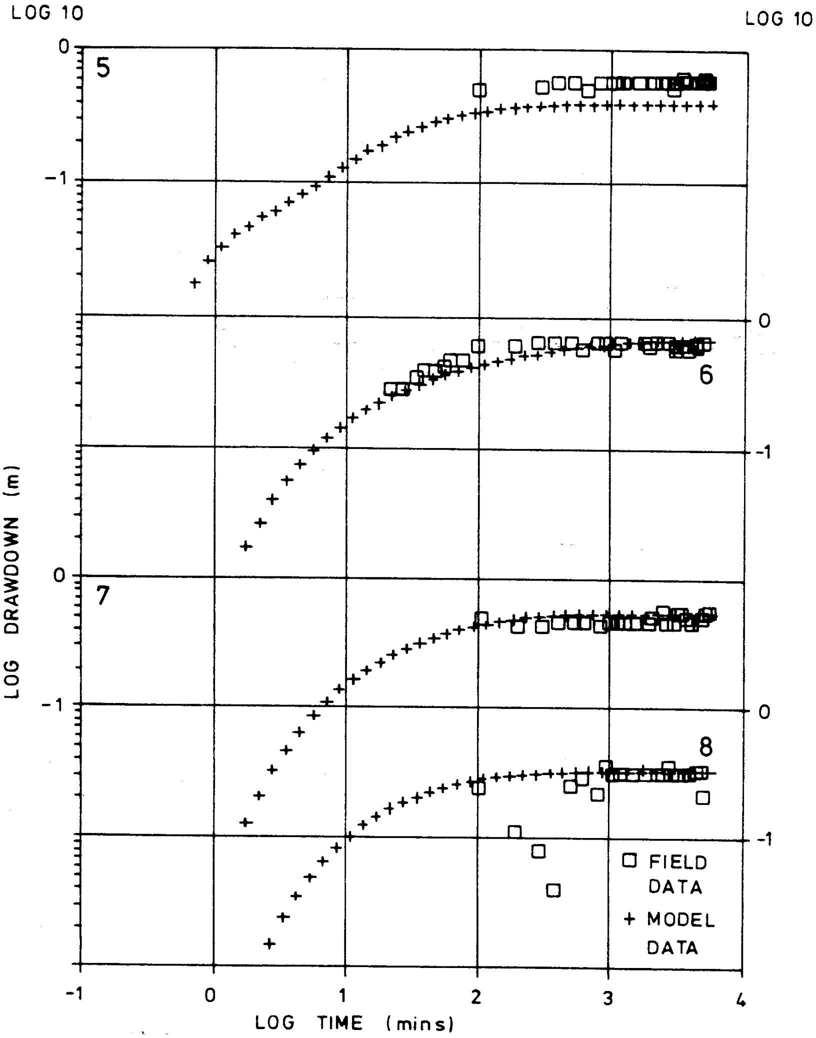


Figure 6 (b) Field and model drawdowns at Pl ports 5-8.

distribution. The non- steady-state model responses at Pl port I to 5 indicate a slight inflexion, probably due to vertical flow from the zone of high groundwater head at the base of the Bunter Pebble Beds.

MODELLED RESULTS FOR WELL W2

The model was also applied to the interpretation of a pumping-test carried out in well W2 (Figures 3 and 4). Details of the test in W2, over a test interval between 50m OD and -3m OD, are given in Figure 8. Model and field drawdown data were compared at the P7 piezometer ports.

The test proved extremely difficult to interpret satisfactorily. While the level in the well fell by 22m during the test, the drawdown at P7 port 5, 21.5m from W2, was only 1.44m. Non-linear well losses were simulated that involved multiplying the permeability in the vicinity of the well by a factor of 0.07. It then became very difficult to force drawdowns of 0.58m and 0.79m at P7 ports 1 and 2, up to 50m below the base of the test interval. Eventually a reasonable solution was obtained that involved a vertical fracture flow mechanism within the Bunter Pebble Beds and Littleworth Sandstones. This is supported by other field evidence that indicates that vertical fracturing, possibly due to deeper mine workings, extend upwards from the Coal Measures into the Triassic strata. Very good non-steady-state agreement was obtained at P7 ports 5 and 7 (Figure 8), with the drawdown-time curve for port 7 showing the characteristics of delayed yield.

Interpretation of data from the two pumping-tests indicates the general increase in permeability of the Bunter Pebble Beds, by an order of magnitude, from the Wl area towards the W2 area. Both permeability distributions indicate that the highest permeability ground exists in the lower Bunter Pebble Beds and the Littleworth Sandstone. The horizon of very low vertical permeability at the top of the Bunter Pebble Beds exist both in the Wl and W2 areas.

DRIFT INFLOW MODELLING

The relationship between the Wl and W2 wells and the proposed drift line is shown in Figure 9. The objective of the inflow modelling was to predict inflows to the drift during construction through the Triassic strata. Inflows to the drift were assessed using the steady-state finite-difference radial (r-z) modelling technique developed by Edwards (1987) and reported in Lloyd and Edwards (1988). In this technique steady-state inflows to a short section of (unlined) open drift are approximated by modelling flow towards a cylindrical opening in a radial (r-z) model. A comparison between three-dimensional and radial (r-z) model results has indicated a relationship, in terms of expected inflow, between the length of drift open section and the size of cylindrical opening in a radial model (Edwards, 1987). Although drift inflows are clearly three-dimensional, this technique provides reasonable results are a fraction of the computer costs required for a three-dimensional model.

Inflows to the drift were evaluated at different levels along the line of construction, based on a 5m open section of 7.5m diameter (Figure 9). The permeability distribution obtained from the Wl pumping-test was employed to a depth of Om OD, and the W2 permeability distribution was employed at deeper levels. The vertical fracture mechanism employed in the W2 data interpretation was employed at all levels in the drift inflow assessment in the W2 area. The inflow predictions in the W2 area are therefore considered to be pessimistic but allow for the uncertainty in the location of fracture systems in the Bunter Pebble Beds and Littleworth Sandstone. These potential inflows exceed 20,000 cu.m/day in the lower Bunter Pebble Beds and the Littleworth Sandstone.

Although only steady-state inflows have been evaluation here, work by Edwards (1987) on non-steady-state shaft inflows has shown that inflows decay at a rate controlled

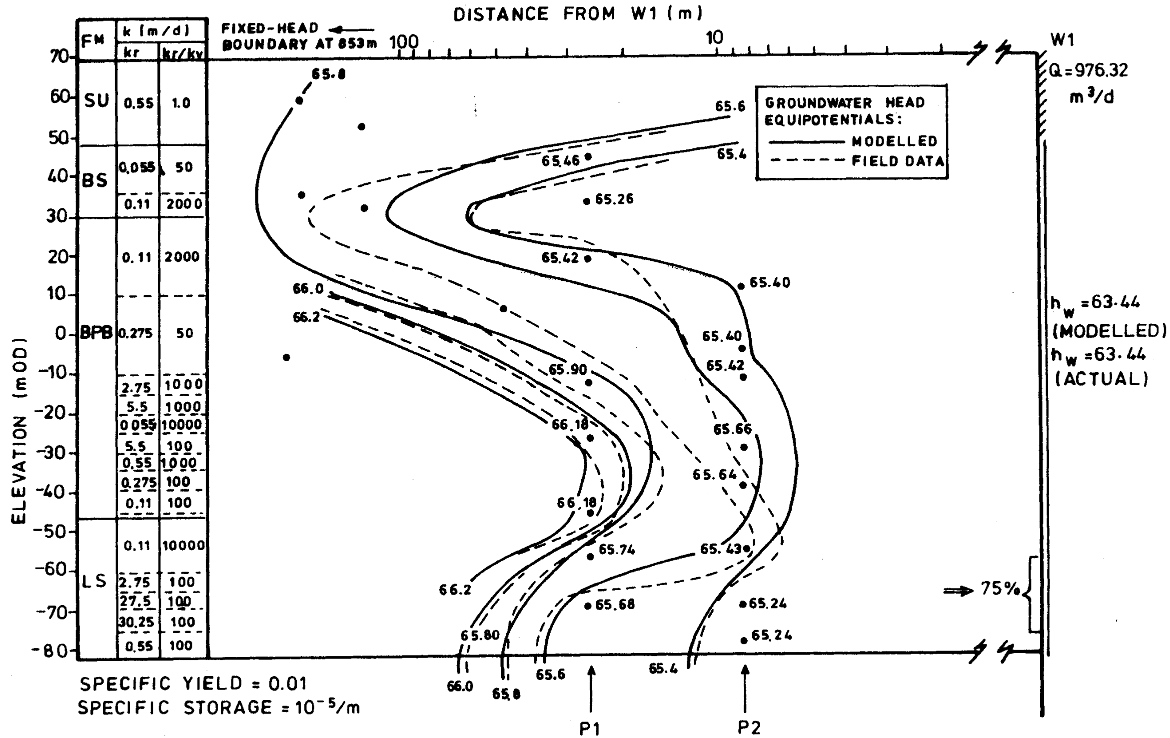


Figure 7. Pumping-test in W1: Comparison between steady-state modelled and field ground- water head distribution. Head values at ports are modelled. Flow arrow indicates modelled inflow distribution to open well.

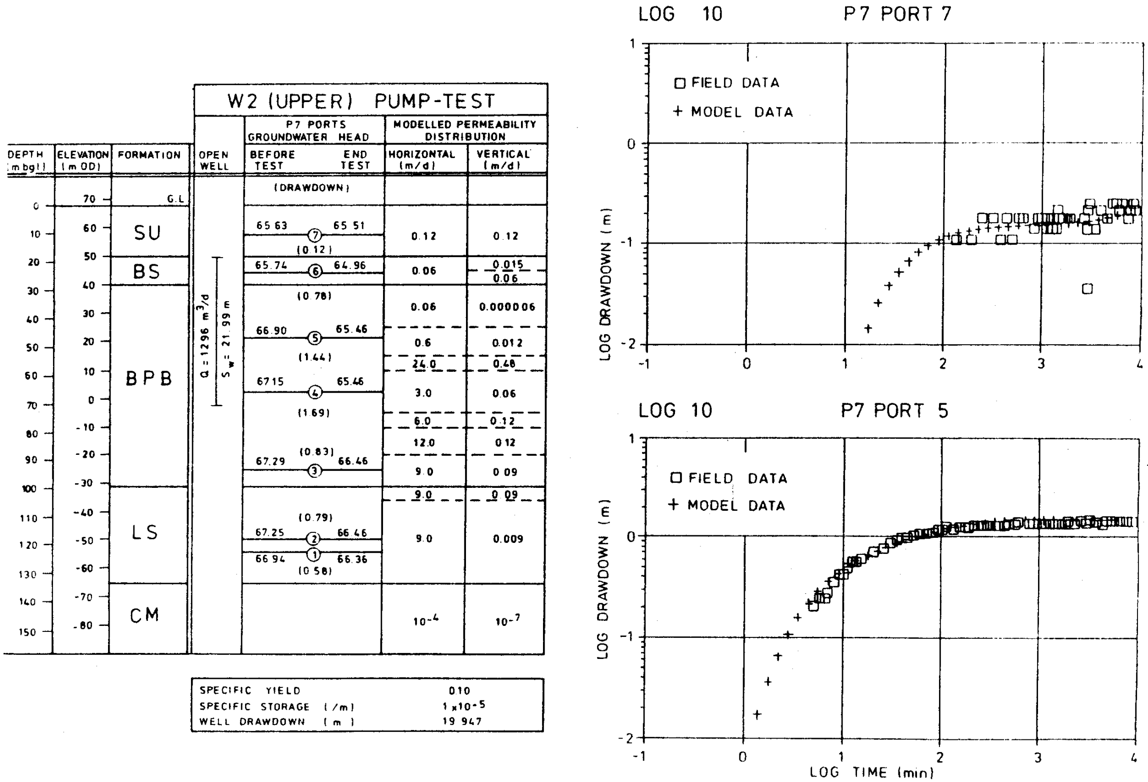


Figure 8. Details and results of the W2 pumping-test interpretation.

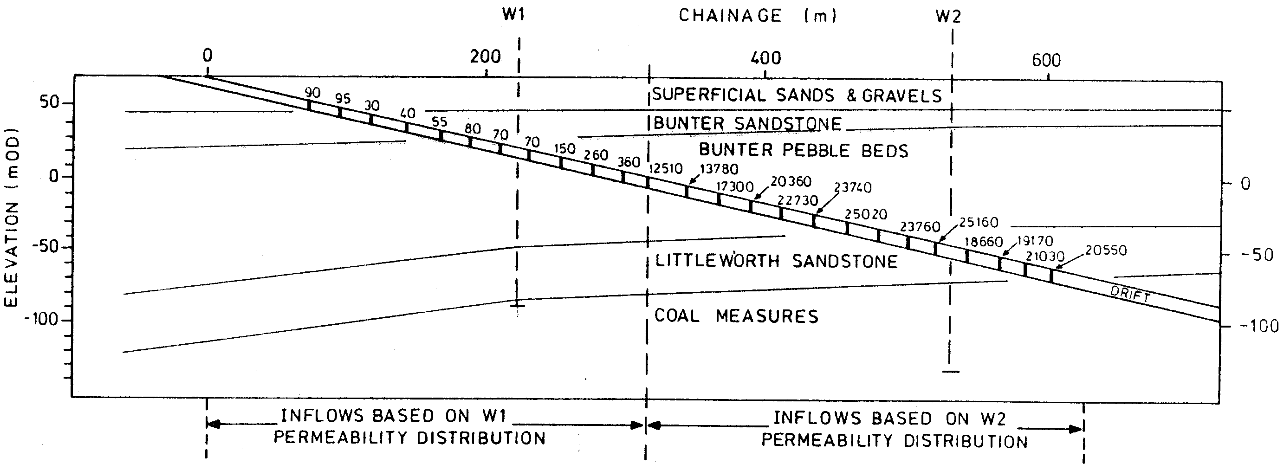


Figure 9. Drift inflow predictions. Inflows are estimated for a Sm open section of drift of 7.5m diameter. Inflow units are cubic metres per day.

mainly by the storage properties of the ground. Where the specific storage coefficient is relatively low (10-6 - 10-7/m), inflows decay quickly to near-steady levels, and steady-state inflow predictions are probably reasonable for much of the construction period. However, where unconfined storage (specific yield) is dominant, inflows will take longer to approach steady values. This will apply above the low vertical permeability zone at the top of the Bunter Pebble Beds, in the Bunter
Sandstone and superficial deposits.

CONCLUSIONS

A radial (r-z-t) model has been developed for the interpretation of multiport piezometer data from pumping-tests carried out in multilayered ground. A detailed approximation of horizontal and vertical permeability conditions can be achieved. Increased sophistication in site investigation techniques is therefore matched by improved methods of data interpretation.

Once the three-dimensional permeability conditions have been approximated, inflows to a proposed drift can be evaluated using a radial (r-z) model. This technique allows not only detailed inflow predictions, but also information on the dominant hydrogeological controls on inflow rates.

Both the pumping-test data interpretation and the inflow modelling described in this paper demonstrate the value of applying numerical modelling techniques to mine inflow assessments. A better understanding of a groundwater system is achieved than could be obtained from conventional analytical techniques, and this optimises decision-making with regard to the economic feasibility of new mine development.

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